Student Number: ___________________

THE KING'S SCHOOL

2022

YEAR 12
Trial

Mathematics Advanced

General Instructions

- Reading Time — 10 minutes
- Working Time — 3 hours
- Write using black pen
- NESA approved calculators may be used
- A formulae and data sheet is provided at the back of this paper

Total Marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II- 90 marks (pages 6–19)

- Attempt Questions 11-30
- Allow about 2 hours and 45 minutes for this section
- Show relevant mathematics reasoning and/or calculations

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is $\int e^{2x} - e \, dx$?

A. $\frac{e^{2x}}{2} - e + c$

B. $\frac{e^{2x}}{2} - ex + c$

C. $2e^{2x} - e + c$

D. $2e^{2x} - ex + c$

2 Which of the following is equal to $\frac{1}{3\sqrt{5}-\sqrt{2}}$?

A. $\frac{3\sqrt{5}-\sqrt{2}}{13}$

B. $\frac{3\sqrt{5}+\sqrt{2}}{13}$

C. $\frac{3\sqrt{5}-\sqrt{2}}{43}$

D. $\frac{3\sqrt{5}+\sqrt{2}}{43}$

3 How many significant figures are there in the first 6 digits of sin45°?

A. 3

B. 4

C. 5

D. 6

4 What is the correct solution to $\log_3(2x + 5) = 3$?

A. 0

B. 1

C. 3

D. 11

5 How many solutions of the equation $(2\cos x)(\tan x - \sqrt{3}) = 0$ lie between 0 and 2π?

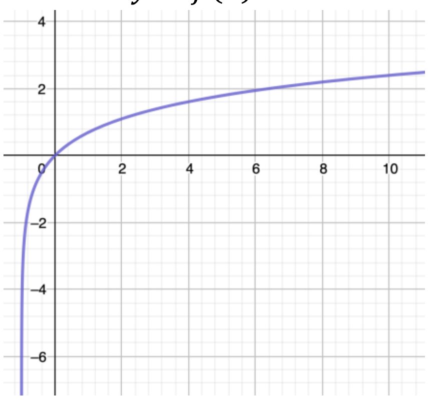
A. 1

B. 2

C. 3

D. 4

6 The graph below shows the function $y = f(x)$.



Which of the following functions best represents $y = f(x)$?

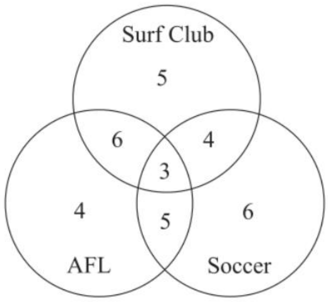
A. $y = \sqrt{x}$

B. $y = \ln(x + 1)$

C. $y = -e^{-x} + 1$

D. $y = -\frac{1}{x+1} + 1$

7 In a classroom, students were asked what sports club they are members of and the results are shown in the Venn diagram below.



A student who is a member of a soccer club is chosen at random.

What is the probability that they are also a member of the surf club?

A. $\frac{2}{9}$

B. $\frac{4}{11}$

C. $\frac{7}{18}$

D. $\frac{2}{5}$

8 Jack invests his money for two years at 2% per half year, compounded every half year.

Compounded values of $1

Period	Interest rate per period				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.010	1.020	1.030	1.040	1.050
2	1.020	1.040	1.061	1.082	1.103
3	1.030	1.061	1.093	1.125	1.158
4	1.041	1.082	1.126	1.170	1.216
5	1.051	1.104	1.159	1.217	1.276
6	1.062	1.126	1.194	1.265	1.340
7	1.072	1.149	1.230	1.316	1.407
8	1.083	1.172	1.267	1.369	1.477

Using the table, which figure should Jack use to calculate his investment?

A. 1.010

B. 1.040

C. 1.041

D. 1.082

9 A box contains n marbles. Let k represent the red marbles and the remaining marbles are blue. Two marbles are chosen from the box.

If the first marble is not replaced after the second is chosen, then what is probability that both marbles are the same colour?

A. $\frac{k^2+(n-k)^2}{n^2}$

B. $\frac{2k(n-k-1)}{n(n-1)}$

C. $\frac{k^2+(n-k-1)^2}{n^2}$

D. $\frac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$

10 The length of a student's haircut is normally distributed with an average of 4.8 mm and a standard deviation of 1.2 mm.

What does a standardised hair length of $z = -0.5$ correspond to?

A. 2.4 mm

B. 3.6 mm

C. 4.2 mm

D. 5.4 mm

END OF SECTION I

Section II

90 marks
Attempt Questions 11 - 30
Allow about 2 hours & 45 minutes for this section

Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks) **3**

Calculate the sum of the arithmetic series $6 + 13 + 20 + \cdots + 1105$.

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Question 12 (4 marks) **4**

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} + \sin^2 x + \cos^2 x \, dx$.

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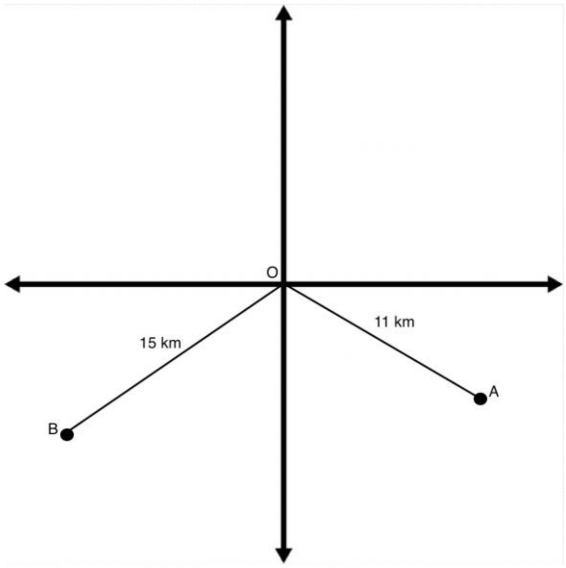
Question 13 (6 marks)

Alex (A) and Beth (B) both start running from a point (O). Alex reaches 11 km on her journey and at a bearing of $110°T$. Beth runs 15 km in the direction of $S40°W$.
This scenario is illustrated in the diagram below.



(a) Calculate the shortest distance between Alex and Beth to the nearest 1 decimal place. **3**

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(b) What is the true bearing of Beth from Alex to the nearest degree? **3**

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Question 14 (3 marks) **3**

Show that when $f(x) = e^x(x^2 + 9)^3$ is differentiated, it becomes

$$f'(x) = e^x(x^2 + 9)^2(x + 3)^2.$$

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Question 15 (3 marks) **3**

James and Kyle are both goal kickers for their respective teams.
The probability that James kicks a goal is $\frac{2}{7}$ and the probability that Kyle kicks a goal is $\frac{5}{8}$.

What is the probability that, on their next 3 attempts at goal, at least one of the them kicks a goal?

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Question 16 (3 marks) **3**

Evaluate $\int_2^3 \frac{6x^2}{x^3-2}\,dx$. Give your answer in exact form.

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Question 17 (7 marks)

Consider the curve $y = x^3 - 12x^2 + 36x$.

(a) Find the x and y intercepts. **2**

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(b) Find any stationary points and determine their nature. **3**

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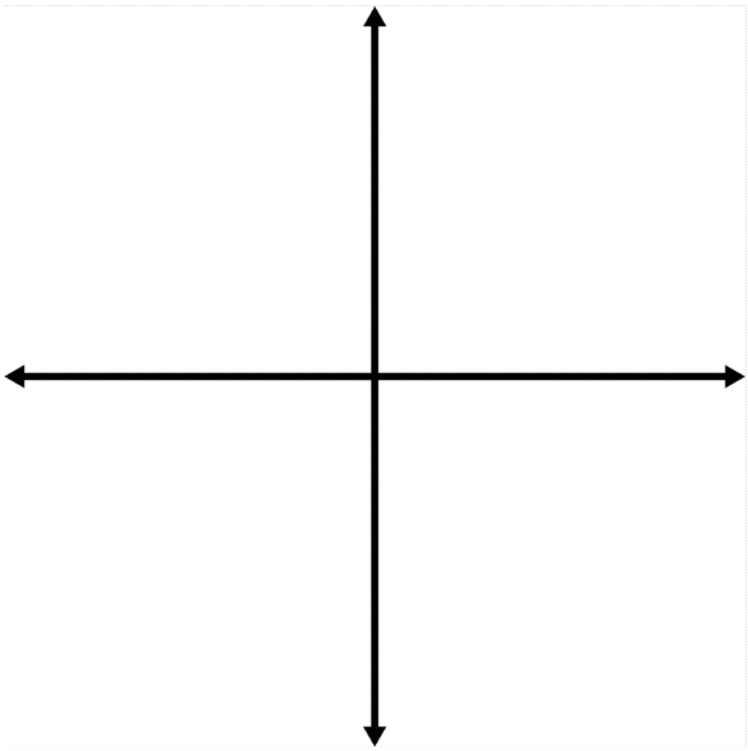
(c) Hence sketch for $-1 \leq x \leq 7$ showing all features from parts (a) and (b). **2**



Question 18 (4 marks) **4**

The population P of a certain bird is decreasing at a rate proportional to P.
That is $\frac{dP}{dt} = -kP$, where k is a positive integer and t is measured in years.

In January 2010, there were 5000 birds and by January 2020, the population had decreased to 3450.

If the population continues to decrease at this rate, what will be the expected population in 2030?

Question 19 (2 marks) **2**

Differentiate ln $(\sin x)$.

Question 20 (2 marks) **2**

The circle $x^2 - 4x + y^2 + 2y - 5 = 0$ is reflected in the y-axis.

Determine the coordinates of the centre and radius of the reflected circle.

Question 21 (9 marks)

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} cosx, & 0 \leq x \leq k \\ 0, & for\ all\ other\ values\ of\ x \end{cases}$$

(a) Find the value of k. **2**

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(b) Determine the mode of X. **2**

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(c) By finding the derivative of $g(x) = xsinx + cosx$, calculate the exact mean of X. **3**

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(d) Find the probability that X is greater than or equal to 2. Give your answer correct to 3 decimal places. **2**

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Question 22 (6 marks)

A cylindrical container closed at both ends is to be made from thin sheet metal. The container is to have a radius of r cm and height of h cm such that the volume is 2000π cm^3.

(a) Show that the area of sheet metal required to make the container is **2**

$$2\pi r^2 + \frac{4000\pi}{r} \text{ cm}^2.$$

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(b) Hence find the minimum area of sheet metal required to make the container. **4**

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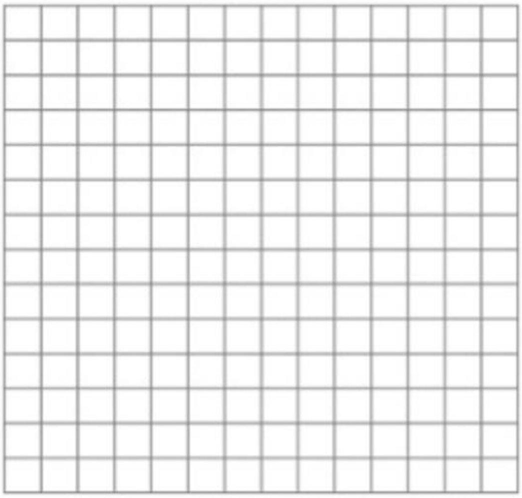
Question 23 (7 marks)

The results of two tests of students were analysed and the data is shown in the table.

Student	1	2	3	4	5	6	7	8
Test 1	28	39	38	30	42	43	33	10
Test 2	12	23	16	16	28	18	24	7

Note: Test 1 was out of 50 and Test 2 was out of 30.

(a) Graph in a scatter plot showing the line of best fit. **3**



(b) There is a student missing from the dataset because he was absent from test 2. **2**

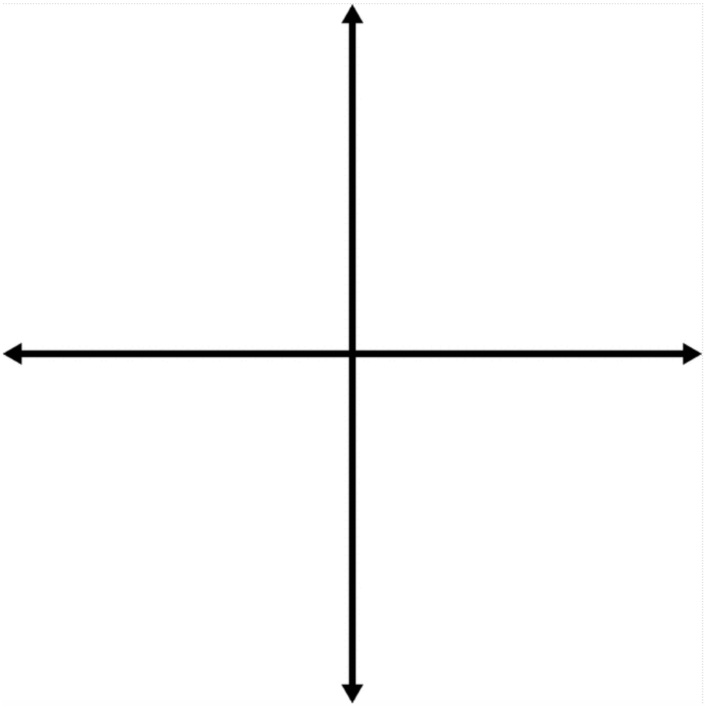
Using the equation of the line of best fit, predict the score for test 2 if his score in test 1 was 30.

(c) Justify whether the prediction is valid or not by calculating Pearson's correlation coefficient. **2**

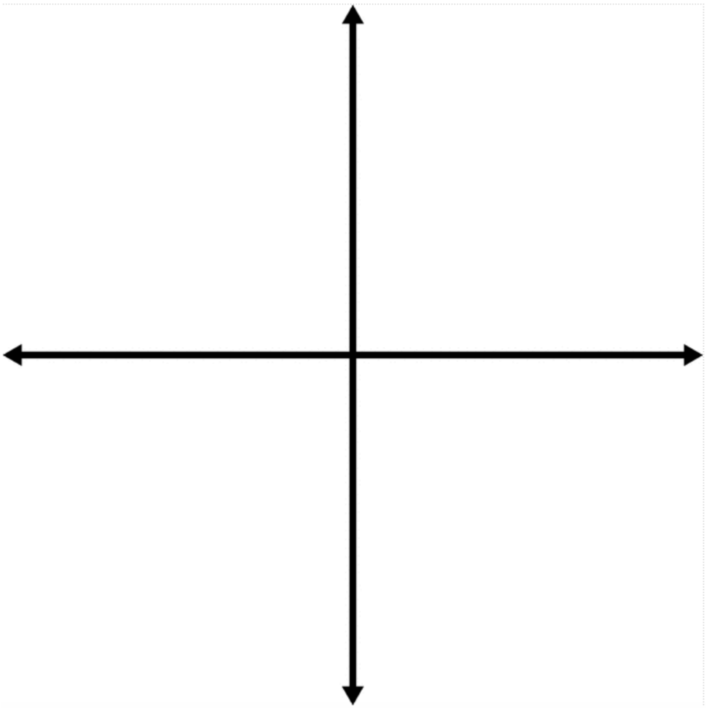
Question 24 (6 marks)

Consider the function $f(x) = -(x+1)^2(x-2)$.

(a) Sketch $y = f(x)$ and $y = 2f(x)$ on the same Cartesian plane below, labelling the coordinates of all intercepts and turning points. **4**



(b) Dilate $y = f(x)$ horizontally by a factor of 2, labelling the coordinates of all intercepts and turning points. **2**



Question 25 (2 marks) 2

A radioactive substance is decaying and releasing radiation.
The amount of radiation R at time t, in years, is given by

$$R = R_o e^{-2.89 \times 10^{-5} t}$$

where R_o is the initial amount of radiation.

Calculate the half-life of the substance, to the nearest 10 years.

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Question 26 (3 marks) 3

Use the Trapezoidal rule with 5 function values to find an approximate value of

$$\int_2^4 \frac{x}{2} \ln x \, dx.$$

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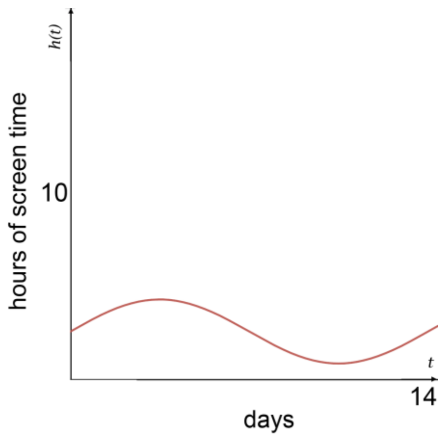
Question 27 (5 marks)

The hours of screen time a student spends in a fortnight can be modelled by

$$h(t) = a\sin\frac{\pi t}{7} + b$$

where t is time in days and $0 \leq t \leq 14$. The hours of screen time reaches a maximum of 5 hours half way through day 3 and a minimum of 1 hour half way through day 10.
The graph of $h(t)$ is shown below.



(a) What are the values of a and b? **1**

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(b) On what days is the screen time increasing by half an hour? **4**

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Question 28 (5 marks)

A particle is moving in a straight line. After time t seconds its displacement x metres from a fixed point O on the line is given by $x = t - 3\ln(t + 1)$. The particle returns to its starting point after T seconds.

(a) Find when the particle is at rest. **1**

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(b) Find in simplest exact form, the distance travelled by the particle in the first T seconds of its motion. **2**

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(c) Show that $e^T = (T + 1)^3$. **2**

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Question 29 (5 marks)

At the start of the year, Joe won a second division lottery prize of $300 000. He decided to put the money into an account that earns interest of 4% p.a. compounded monthly.
Joe then uses the money from this account to pay off $25 000 at the end of each year to service a current loan.
Let A_n dollars denote the amount remaining in his account after the n^{th} year.

(a) Find an expression for the amount of money remaining after the first 3 years. **2**

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(b) After 10 years, Joe paid off his loan in full and wants to purchase a car for $100 000. Write down an expression of A_n and hence determine whether Joe can afford this car. **3**

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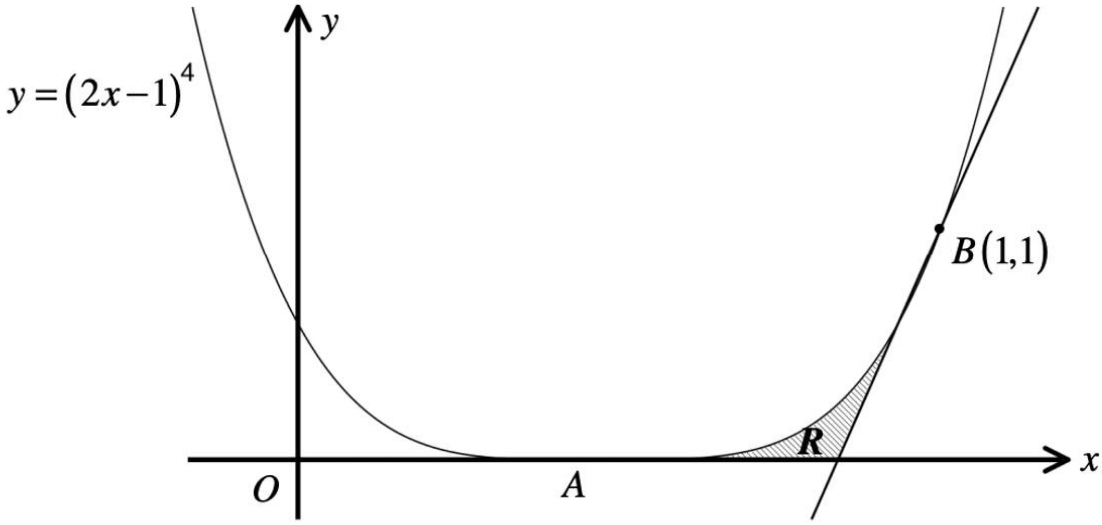
Question 30 (5 marks) **5**

The figure below shows region R bound by a curve, a tangent to the curve at B and the x-axis.



Calculate the area of region R.

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End of Section II

End of Paper

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is $\int e^{2x} - e\,dx$?

A. $\frac{e^{2x}}{2} - e + c$

B. $\frac{e^{2x}}{2} - ex + c$

C. $2e^{2x} - e + c$

D. $2e^{2x} - ex + c$

2 Which of the following is equal to $\frac{1}{3\sqrt{5}-\sqrt{2}}$?

A. $\frac{3\sqrt{5}-\sqrt{2}}{13}$

B. $\frac{3\sqrt{5}+\sqrt{2}}{13}$

C. $\frac{3\sqrt{5}-\sqrt{2}}{43}$

D. $\frac{3\sqrt{5}+\sqrt{2}}{43}$

3 How many significant figures are there in the first 6 digits of sin45°?

A. 3

B. 4

C. 5

D. 6

4 What is the correct solution to $\log_3(2x + 5) = 3$?

A. 0

B. 1

C. 3

D. 11

5 How many solutions of the equation $(2\cos x)(\tan x - \sqrt{3}) = 0$ lie between 0 and 2π?

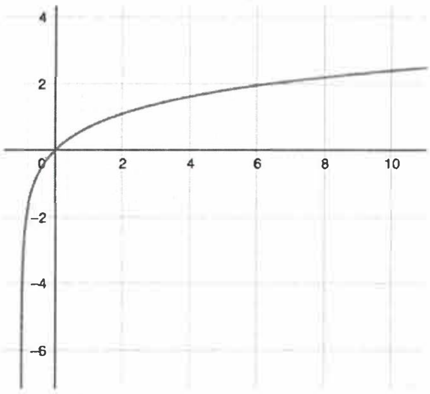
A. 0

B. 1

C. 2

D. 3

6 The graph below shows the function $y = f(x)$.



Which of the following functions best represents $y = f(x)$?

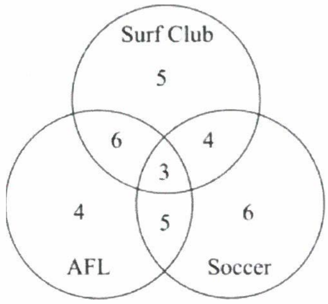
A. $y = \sqrt{x}$

B. $y = \ln(x + 1)$

C. $y = -e^{-x} + 1$

D. $y = -\frac{1}{x+1} + 1$

7 In a classroom, students were asked what sports club they are members of and the results are shown in the Venn diagram below.



A student who is a member of a soccer club is chosen at random.

What is the probability that they are also a member of the surf club?

A. $\frac{2}{5}$

B. $\frac{4}{11}$

C. $\frac{2}{9}$

D. $\frac{7}{18}$

8 Jack invests his money for two years at 2% per half year, compounded every half year.

Compounded values of $1

Period	Interest rate per period				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.010	1.020	1.030	1.040	1.050
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7	1.072	1.149	1.230	1.316	1.407
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Using the table, which figure should Jack use to calculate his investment?

A. 1.010

B. 1.040

C. 1.041

D. 1.082

9 A box contains n marbles. Let k represent the red marbles and the remaining marbles are blue. Two marbles are chosen from the box.

If the first marble is not replaced after the second is chosen, then what is probability that both marbles are the same colour?

A. $\frac{k^2+(n-k)^2}{n^2}$

B. $\frac{2k(n-k-1)}{n(n-1)}$

C. $\frac{k^2+(n-k-1)^2}{n^2}$

D. $\frac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$

10 The length of a student's haircut is normally distributed with an average of 4.8 mm and a standard deviation of 1.2 mm.

What does a standardised hair length of $z = -0.5$ correspond to?

A. 2.4 mm

B. 3.6 mm

C. 4.2 mm

D. 5.4 mm

END OF SECTION I

Section II

90 marks
Attempt Questions 11 - 30
Allow about 2 hours & 45 minutes for this section

Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks) 3

Calculate the sum of the arithmetic series $6 + 13 + 20 + \cdots + 1105$.

$1105 = 6 + (n-1) \times 7$ $a = 6$ $d = 7$

$1099 = 7n - 7$ $\therefore S_{158} = 79(6 + 1105)$

$1106 = 7n$ $= 87769$

$n = 158$

Question 12 (4 marks) 4

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} + \sin^2 x + \cos^2 x \, dx$.

$\int_{\pi/6}^{\pi/4} \tan^2 x + 1 \, dx = \int_{\pi/6}^{\pi/4} \sec^2 x \, dx$

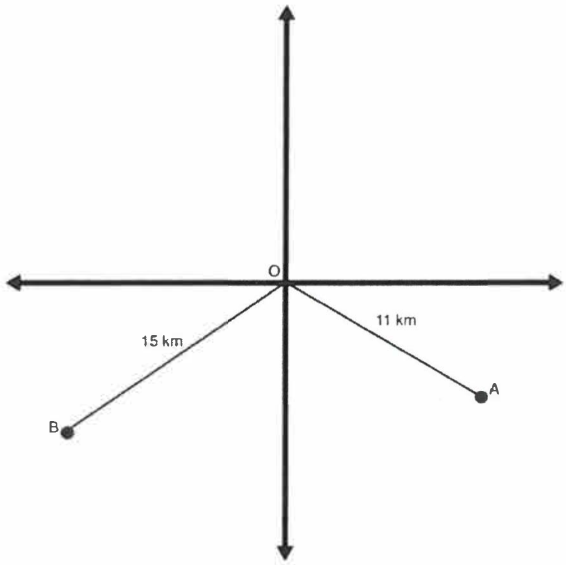
$= [\tan x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= [1] - [\frac{1}{\sqrt{3}}]$

$= 0.42$

Question 13 (6 marks)

Alex (A) and Beth (B) both start running from a point (O). Alex reaches 11 km on her journey and at a bearing of $110°T$. Beth runs 15 km in the direction of $S40°W$.
This scenario is illustrated in the diagram below.



(a) Calculate the shortest distance between Alex and Beth to the nearest 1 decimal place. **3**

$d = \sqrt{15^2 + 11^2 - 2 \times 15 \times 11 \times \cos 110}$

$= 21.4\text{km}$

(b) What is the true bearing of Beth from Alex to the nearest degree? **3**

$\frac{\sin\theta}{15} = \frac{\sin 110}{21.4}$

$\theta = \sin^{-1}\left(\frac{15 \sin 110}{21.4}\right)$

$= 41°''$

$\therefore 360 - 70 - 41°''$

$= 249°$

Question 14 (3 marks) 3

Show that when $f(x) = e^x(x^2+9)^3$ is differentiated, it becomes

$$f'(x) = e^x(x^2+9)^2(x+3)^2.$$

$u' = e^x$
$v' = 6x(x^2+9)^2$

$f'(x) = e^x 6x(x^2+9) + e^x(x^2+9)^3$

$= e^x(x^2+9)^2(6x + x^2 + 9)$

$= e^x(x^2+9)^2(x+3)^2$

Question 15 (3 marks) 3

James and Kyle are both goal kickers for their respective teams.
The probability that James kicks a goal is $\frac{2}{7}$ and the probability that Kyle kicks a goal is $\frac{5}{8}$.

What is the probability that, on their next 3 attempts at goal, at least one of the them kicks a goal?

$P(\text{Both Kick}) = \frac{2}{7} \times \frac{5}{8}$

$= \frac{5}{28}$

$\therefore P(\text{Both Miss}) = \frac{5}{7} \times \frac{3}{8}$

$= \frac{15}{56}$

$\therefore 1 - P(\text{Both miss})^3$

$= 0.98$

Question 16 (3 marks) 3

Evaluate $\int_2^3 \frac{6x^2}{x^3-2}\, dx$. Give your answer in exact form.

let $x^3 - 2 = u$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int_2^3 \frac{2\,6x^2}{u} \cdot \frac{du}{3x^2} = \int_2^3 \frac{2}{u}\, du$

$= 2\ln|u|$

$= 2\ln 25 - 2\ln 6$

$= \ln \frac{625}{36}$

Question 17 (7 marks)

Consider the curve $y = x^3 - 12x^2 + 36x$.

(a) Find the x and y intercepts. **2**

y-Int = 0 $0 = x(x^2 - 12x + 36)$

$0 = x(x-6)^2$

$x = 0, 6$

(b) Find any stationary points and determine their nature. **3**

$y' = 3x^2 - 24x + 36 \Rightarrow 0 = x^2 - 8x + 12$

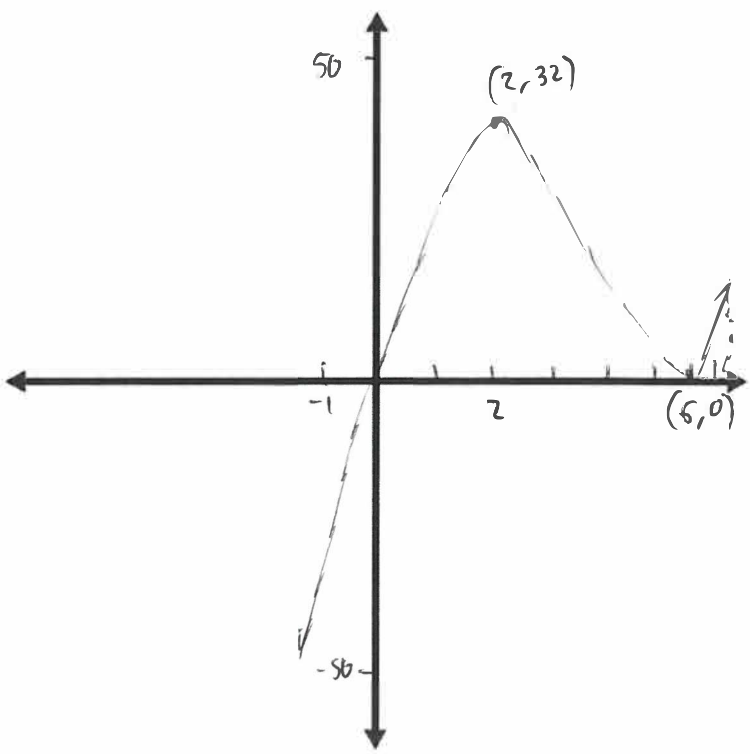
$y'' = 6x - 24$ $0 = (x-6)(x-2)$

$= 6(2) - 24 = -12$ (max) $x = 2, 6$

$= 6(6) - 24 = 12$ (min)

$\therefore (2, 32)$ max & $(6, 0)$ min.

(c) Hence sketch for $-1 \leq x \leq 7$ showing all features from parts (a) and (b). **2**



Question 18 (4 marks) 4

The population P of a certain bird is decreasing at a rate proportional to P.
That is $\frac{dP}{dt} = -kP$, where k is a positive integer and t is measured in years.

In January 2010, there were 5000 birds and by January 2020, the population had decreased to 3450.

If the population continues to decrease at this rate, what will be the expected population in 2030?

spacing

$3450 = 5000e^{-k \times 10}$

$0.69 = e^{-10k}$

$\frac{\ln 0.69}{-10} = k$

$\therefore P = 5000e^{-\left(\frac{\ln 0.69}{-10}\right) \times 20}$

≈ 2380

Question 19 (2 marks) 2

spacing

Differentiate ln (sin$x$).

$y' = \cos x \times \frac{1}{\sin x}$

$= \cot x$

Question 20 (2 marks) 2

The circle $x^2 - 4x + y^2 + 2y - 5 = 0$ is reflected in the y-axis.

Determine the coordinates of the centre and radius of the reflected circle.

spacing

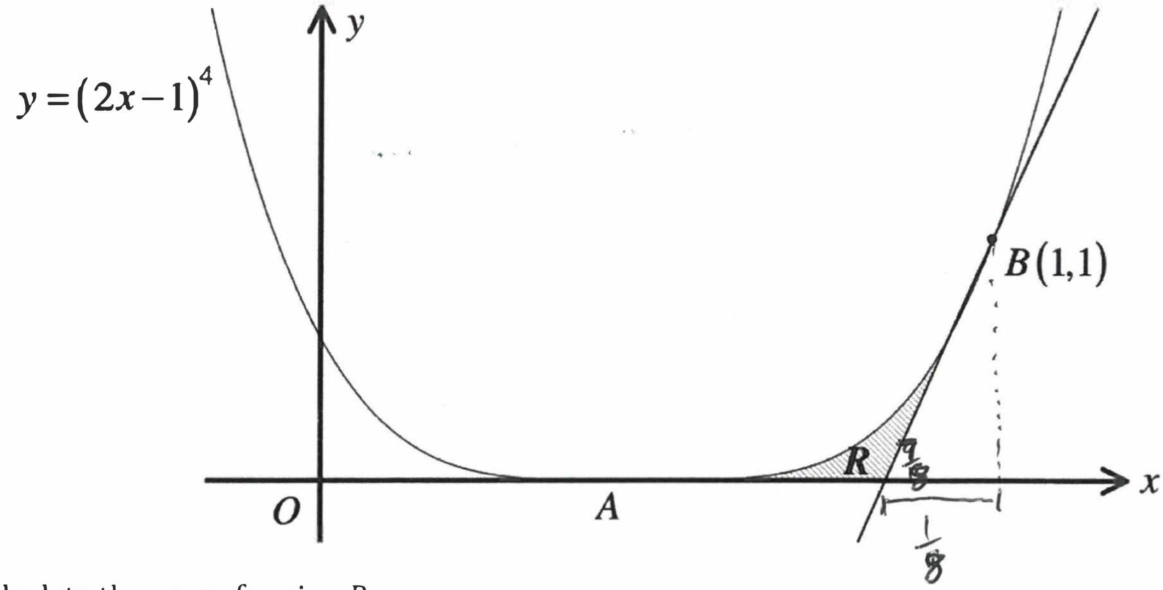
$x^2 - 4x + 4 + y^2 + 2y + 1 = 5 + 4 + 1$

$(x-2)^2 + (y+1)^2 = 10$ $\quad c = 2, -1 \quad r = \sqrt{10}$

$c = (-2, -1) \quad r = \sqrt{10}$

Question 30 (5 marks) **5**

The figure below shows region R bound by a curve, a tangent to the curve at B and the x-axis.



Calculate the area of region R.

$y' = 8(2x-1)^3$ at $(1,1)$

$m = 8$

$\therefore$ tangent B is on $y - 1 = 8(x-1)$

$y = 8x - 7$

$0 = 8x - 7$

$7 = 8x$

$x = \frac{7}{8}$

Area of Triangle

$\frac{1}{8} \times 1 \times \frac{1}{2} = \frac{1}{16}$

$\therefore \int_{\frac{1}{2}}^{1} (2x-1)^4 \, dx - \frac{1}{16}$

$\left[\frac{(2x-1)^5}{10}\right]_{\frac{1}{2}}^{1} - \frac{1}{16}$

$\left[\frac{1}{10}\right] - [0] - \frac{1}{16}$

$= \frac{3}{80} \, u^2$

Question 2 (9 marks)

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} cosx, & 0 \le x \le k \\ 0, & for\ all\ other\ values\ of\ x \end{cases}$$

(a) Find the value of k. **2**

$\int_0^k cosx\,dx = 1$ $\quad [\sin x]_0^k = 1$

$[\sin k] - [\sin 0] = 1$

$\sin k = 1$

$k = \frac{\pi}{2}$

(b) Determine the mode of X. **2**

$f'(x) = -\sin x$ $\quad$ Test 0 to $\frac{\pi}{2}$

$f''(x) = -\cos x$ $\therefore$ max $\quad \cos[0] = 1 \quad \cos[\frac{\pi}{2}] = 0$

$\therefore$ Mode is 0

(c) By finding the derivative of $g(x) = xsinx + cosx$, calculate the exact mean of X. **3**

$g'(x) = x\cos x + \sin x - \sin x$

$= x\cos x \Rightarrow x\,p\,x$

$\int_0^{\frac{\pi}{2}} x\cos x = [x\sin x + \cos x]_0^{\frac{\pi}{2}}$

$= [\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}] - [0\sin 0 + \cos 0]$

$= \frac{\pi}{2} - 1$

(d) Find the probability that X is greater than or equal to 2. Give your answer correct to 3 decimal places. **2**

$1 - \int_0^2 cosx\,dx$ $\quad 1 - [\sin 2] - [\sin 0]$

$1 - [\sin x]_0^2$ $\quad = 0.091$

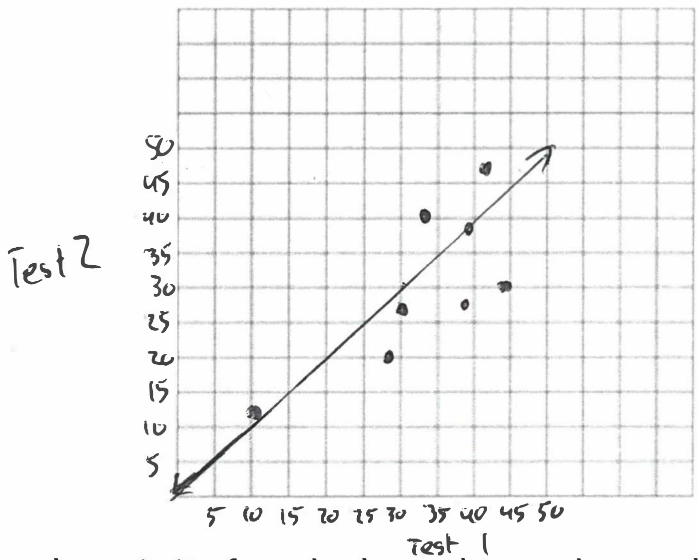
Question 23 (7 marks)

The results of two tests of students ~~was~~ were analysed and the data is shown in the table.

Student	1	2	3	4	5	6	7	8
Test 1	28	39	38	30	42	43	33	10
Test 2 ÷ 30 × 50	12 /20	23 /38 1/3	16 /26 2/3	16 /26 2/3	28 /46 2/3	18 /30	24 /40	7 /11 2/3

Note: Test 1 was out of 50 and Test 2 was out of 30.

(a) Graph in a scatter plot showing the line of best fit. **3**



(b) There is a student missing from the dataset because he was absent from test 2. **2**

Using the equation of the line of best fit, predict the score for test 2 if his score in test 1 was 30.

2.838173629

$y = 0.8252152508x +$

$y = 0.8252152508(30) + 2.83817329$

$= 27.5296315 \left(\times \frac{3}{5}\right)$

$\boxed{= 17}$

(c) Justify whether the prediction is valid or not by calculating Pearson's correlation coefficient. **2**

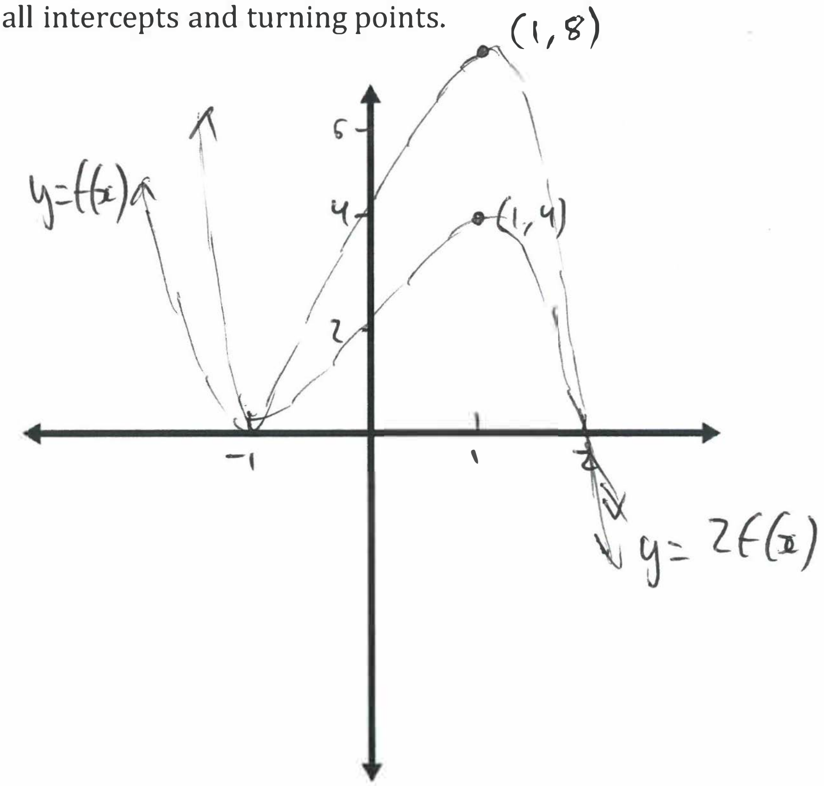
r = 0.78, strong positive correlation

∴ valid.

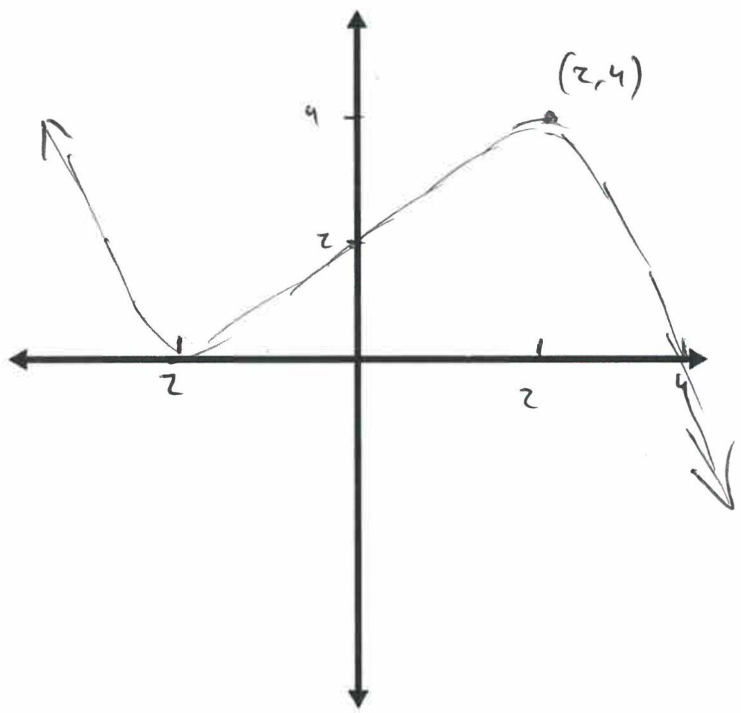
Question 24 (6 marks)

Consider the function $f(x) = -(x+1)^2(x-2)$.

(a) Sketch $y = f(x)$ and $y = 2f(x)$ on the same Cartesian plane below, labelling the coordinates of all intercepts and turning points. **4**



(b) Dilate $y = f(x)$ horizontally by a factor of 2, labelling the coordinates of all intercepts and turning points. **2**



Question 25 (2 marks) 2

A radioactive substance is decaying and releasing radiation.
The amount of radiation R at time t, in years, is given by

$$R = R_o e^{-2.89 \times 10^{-5} t}$$

Where R_o is the initial amount of radiation.

Calculate the half-life of the substance, to the nearest 10 years.

$\frac{1}{2} = e^{-2.89 \times 10^{-5} t}$

$\frac{\ln \frac{1}{2}}{-2.89 \times 10^{-5}} = t$

$t = 23980$

Question 26 (3 marks) 3

Use the Trapezoidal rule with 5 function values to find an approximate value of

$$\int_2^4 \frac{x}{2} \ln x \, dx.$$

x	2	2.5	3	3.5	4
y	ln2	1.145	1.648	2.192	ln16

$\approx \frac{0.5}{2}\left[\ln 2 + \ln 16 + 2(1.145 + 1.648 + 2.192)\right]$

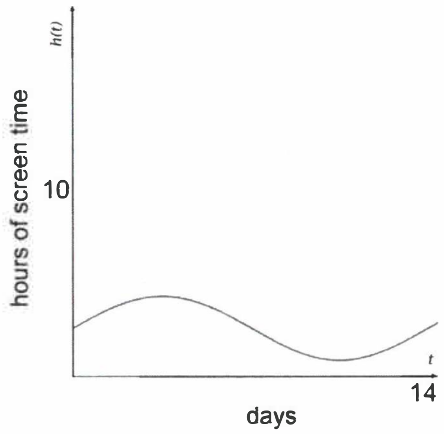
$\approx 3.359 \, u^2$

Question 27 (5 marks)

The hours of screen time a student spends in a fortnight can be modelled by

$$h(t) = a\sin\frac{\pi t}{7} + b$$

where t is time in days and $0 \leq t \leq 14$. The hours of screen time reaches a maximum of 5 hours half way through day 3 and a minimum of 1 hour half way through day 10.
The graph of $h(t)$ is shown below.



(a) What are the values of a and b? **1**

$\frac{5-1}{2}$ $\frac{5+1}{2}$

$a = 2$ $b = 3$

(b) What days ~~is~~ are the screen time increasing by half an hour? **4**

$h'(t) = \frac{2\pi}{7}\cos\left(\frac{\pi t}{7}\right)$

$0.5 = \frac{2\pi}{7}\cos\left(\frac{\pi t}{7}\right)$

$\frac{7}{4\pi} = \cos\frac{\pi t}{7}$

$\cos^{-1}\left(\frac{7}{4\pi}\right) = \frac{\pi t}{7}$

&

$2\pi - \cos^{-1}\left(\frac{7}{4\pi}\right)$

$\therefore$ when $h'(t) = 0.5$

$t = 7\cos^{-1}\left(\frac{7}{4\pi}\right)$

$= 2.18$, Day 2

&

$\frac{7\left(2\pi - \cos^{-1}\left(\frac{7}{4\pi}\right)\right)}{\pi}$

$= 11.8$, Day 11

Question 28 (5 marks)

A particle is moving in a straight line. After time t seconds its displacement x metres from a fixed point O on the line is given by $x = t - 3\ln(t + 1)$. The particle returns to its starting point after T seconds.

(a) Find when the particle is at rest. **1**

$\dot{x} = 1 - \frac{3}{t+1}$ $\quad 1 = \frac{3}{t+1}$

$t + 1 = 3$ $\quad \therefore t = 2$

(b) Find in simplest exact form, the distance travelled by the particle in the first T seconds of its motion. **2**

Initially at O then at $t = 2 \Rightarrow |2 - 3\ln 3|$

then returns back to O after T seconds

$\therefore 2 \times |(2 - 3\ln 3)|$

$= 2(3\ln 3 - 2)$ m

(c) Show that $e^T = (T + 1)^3$. – full stop **2**

$0 = T - 3n(T+1) \quad \therefore T = 3\ln(T+1)$

$= \ln(T+1)^3$

$e^T = (T+1)^3$

Question 29 (5 marks)

At the start of the year, Joe won a second division lottery prize of \$300 000. He decided to put the money into an account that earns interest of 4% p.a. compounded monthly.
Joe then uses the money from this account to pay off \$25 000 at the end of each year to service a current loan.
Let A_n dollars denote the amount remaining in his account after the n^{th} year.

(a) Find an expression for the amount of money remaining after the first 3 years. **2**

$A_1 = 300000 \times (1.00\dot{3})^{12} - 25000$

$A_2 = A_1 (1.00\dot{3})^{12} - 25000$

$= [300000 (1.00\dot{3})^{12} - 25000] (1.00\dot{3})^{12} - 25000$

$= 300000 ((1.00\dot{3})^{12})^2 - 25000 (1.00\dot{3}^{12}) - 25000$

$A_3 = A_2 (1.00\dot{3}^{12}) - 25000$

$= [300000 (1.00\dot{3}^{12})^2 - 25000 (1.00\dot{3}^{12}) - 25000] (1.00\dot{3}^{12}) - 25000$

$= 300000 (1.00\dot{3}^{12})^3 - 25000 [(1.00\dot{3}^{12})^2 + (1.00\dot{3}^{12}) + 1]$

(b) After 10 years, Joe paid off his loan in full and wants to purchase a car for \$100 000. Write down an expression of A_n and hence determine whether Joe can afford this car. **3**

$A_n = 300000 (1.00\dot{3}^{12})^n - 25000 [1 + (1.00\dot{3}^{12}) + (1.00\dot{3}^{12})^2 \dots + (1.00\dot{3}^{12})^{n-1}]$

$= 300000 (1.00\dot{3}^{12})^n - 25000 \times \dfrac{(1.00\dot{3}^{12})^n - 1}{1.00\dot{3}^{12} - 1}$

$A_{10} = 300000 \times (1.00\dot{3})^{120} - 25000 \times \dfrac{(1.00\dot{3})^{120} - 1}{(1.00\dot{3})^{12} - 1}$

$= 146062.95$

$\therefore$ Can afford car.

Question ~~30~~ 22 (6 marks)

A cylindrical container closed at both ends is to be made from thin sheet metal.
The container is to have a radius of r cm and height of h cm such that the volume is 2000π cm^3.

(a) Show that the area of sheet metal required to make the container is **2**

$$2\pi r^2 + \frac{4000\pi}{r} \text{ cm}^2.$$

$V = \pi r^2 h$

$2000\pi = \pi r^2 h$

$h = \frac{2000}{r^2}$

$\therefore SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r\left(\frac{2000}{r^2}\right)$

$= 2\pi r^2 + \frac{4000\pi}{r}$ as required.

(b) Hence find the minimum area of sheet metal required to make the container. **4**

$SA' = 4\pi r - \frac{4000\pi}{r^2}$

$0 = 4\pi r - \frac{4000}{r^2}$

$\frac{4000\pi}{r^2} = 4\pi r$

$4000\pi = 4\pi r^3$

$1000 = r^3$

$r = 10$

$SA'' = 4\pi + \frac{8000\pi}{r^3}$

for $r = 10$: $4\pi + \frac{8000\pi}{(10)^3}$

$= 12\pi \therefore$ Min

$\therefore SA = 2\pi(10)^2 + \frac{4000\pi}{10}$

$= 600\pi$ or 1884.96 cm^2

End of Section II

End of Paper